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ENRICHING THE ELEMENTARY SCHOOL MATHEMATICS PROGRAM FOR MORE CAPABLE CHILDREN

J. Fred Weaver

Cleo Fisher Brawley

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MATHEMATICALLY SUPERIOR CHILDREN IN THE ELEMENTARY SCHOOL

For understandable reasons there is considerable current interest in providing more effectively for elementary school children who have special abilities in the field of mathematics. In the past all too many of our instructional programs simply have not met the needs of these children to an acceptable extent.

The authors of this issue of the Journal have sought to present some of the major issues associated with this important problem, to advance a profitable approach that may be taken in our efforts to improve upon the existing situation, and to illustrate specifically one particular dimension of that approach. The resulting monograph should be of interest and value to persons in a variety of educational positions, and especially so to the classroom teacher.

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Cleo Fisher Brawley is a Lecturer on Education in the Boston University School of Education. She is concerned principally with the training of prospective elementary school teachers in the area of mathematics, and also has worked closely with some of the teachers-in-service whose theses formed the basis for Part II of this Journal issue.

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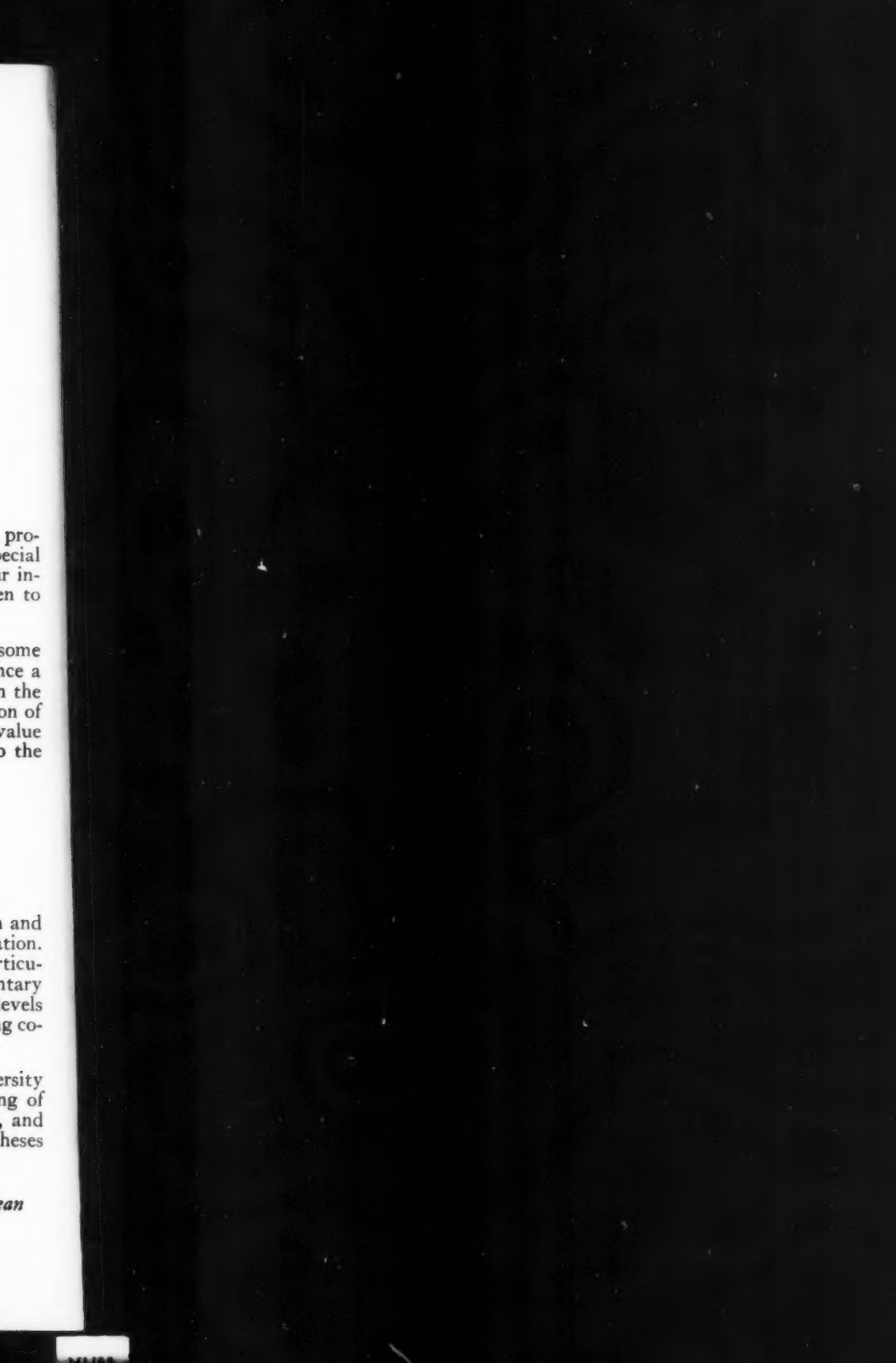

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Enriching the Elementary School Mathematics Program for More Capable Children

J. FRED WEAVER

CLEO FISHER BRAWLEY

TABLE OF CONTENTS

Introduction 2

Part I. Background Considerations 3

Part II. Selected Self-Administering Paper-and-Pencil Enrichment Activities 14

Bibliography 35

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INTRODUCTION

During the past few years several groups of graduate students in the Boston University School of Education have worked on a problem of increasing concern in the field of mathematics instruction at the elementary school level. In general this work focused on the problem of providing effectively for the mathematically superior, or more capable, child. Beginning in 1955–56 with a master's thesis by Yood (A-4)*and continuing in subsequent years with theses by Kling and Triebs (A-3), by Barrett et al. (A-2), and by Banks et al. (A-1), the focus of work became progressively sharper on one particular aspect of the broad problem: the development of self-administering pencil-and-paper mathematical enrichment activities for more capable elementary school children.

All too often materials such as these appear only in unpublished form, known to and used by relatively few persons. It is hoped that in this instance, however, the present publication will help to remedy such a situation.

This monograph has been prepared in two major sections. Part I is devoted to a discussion of pertinent "Background Considerations" relating to the problem of providing effectively for the more capable child, mathematically, at the elementary school level. Part II is composed of "Selected Self-Administering Pencil-and-Paper Enrichment Activities" for mathematically superior elementary school children. These have been chosen from among the activities prepared or suggested in the four theses mentioned previously.

*This and similar designations refer to the bibliography at the end of Part II of this monograph, pp. 35–40.

PART I

BACKGROUND CONSIDERATIONS

A New Problem? Judging from the amount of attention devoted to the matter in our professional literature, one might think that providing effectively for the mathematically superior child in the elementary school is a rather recent problem. During the past five or six years numerous discussions relating to various aspects of this problem have been published in one form or another. Prior to about 1953–54, however, relatively little appeared in print regarding effective provisions for the more capable child in relation specifically to the elementary school mathematics program.

In reality, of course, this is not a new problem at all. Nor is it a separate and discrete problem in the sense that some persons unwittingly lead us to believe. Rather, it is an issue that has been with us a long time as an integral part of the broad problem of providing for individual differences among elementary school children, particularly in relation to their arithmetic or mathematical abilities. Unfortunately, however, until recently we have tended to concentrate most of our instructional attention on average and below average learners, at the expense of more capable learners. These latter pupils have become the "neglected children" of all too many arithmetic programs — children for whom no *effective* special provisions were made.

The reasons for our current interest in the mathematically superior elementary school child are well known and need not be discussed here. It is fine, on the one hand, that we at last have awakened to the particular needs of this neglected group and are seeking to provide for them much more effectively. We must be cautious, on the other hand, and not let our present concern for this group result in a comparative neglect of average and less capable learners. We must strive for a *well balanced* program of provision for individual differences in mathematical ability that will meet effectively the needs of *all* elementary school children. Using the provocative title of Phillips' discussion, we may say: "One Classroom, with Arithmetic and Justice for All" (B-39).

Who Are the More Capable Children, Mathematically? It is quite obvious that before effective provisions can be made for the mathematically superior elementary school child, he must be recognized or identified as such. Obvious though it may be, this fact needs special emphasis.

There is good reason to believe that frequently in the past we have not recognized the actual or potential mathematical superiority of some elementary school children. In some instances we have not recognized this superiority at all. In other instances we have recognized it ultimately, but not soon enough. In either event, our failure to identify those children who are among the more capable mathematically has meant that we have made no effective provision for them. They soon develop undesirable negative attitudes toward mathematics, even to the point of trying to avoid mathematics in every way possible. When the potential of these children finally is recognized (if it is recognized at all), it often then is too late. The damage has been done and cannot be rectified.

Somewhat unfortunately, there is no infallible single criterion or measure of the mathematically superior elementary school child. Rather, he must be recognized on the basis of a variety of factors, each of which is not sufficient in itself to identify him as a more capable child mathematically. Significant distinguishing characteristics of the mathematically superior elementary school child have been indicated in one way or another in connection with discussions by persons such as Brydegaard (B-5), Hlavaty (B-22), Junge (B-27), McSwain and Cooke (B-31), and Weaver (B-46). However, it will be well for us to review certain of these traits in relation to the present monograph.

Objective measures of intelligence and arithmetic achievement are, without doubt, important factors to be considered in identifying the mathematically superior child in the elementary school. A child with high general intelligence and a high level of arithmetic achievement as measured by standardized tests frequently is mathematically talented — *but not necessarily so.* Furthermore, an elementary school child truly may be talented mathematically but may not be outstanding in his performance on standardized tests of general intelligence and arithmetic achievement. Table I which follows may serve to illustrate some of the uncertainties faced by a teacher in identifying mathematically superior children on the basis of their general intelligence and arithmetic achievement.

TABLE I

DISTRIBUTION OF PUPILS IN A SIXTH-GRADE CLASS IN RELATION TO IQ AND ARITHMETIC ACHIEVEMENT

IQ Group	*Number of Pupils in Comparable Group in*				
	*Both AR and AC**	*AR, but not AC*	*AC, but not AR*	*Neither AR nor AC*	*Total*
Upper 27%: N = 8 (IQ = 113 and above)	4	2	1	1	8
Middle 46%: N = 13 (IQ = 93–112)	6	2	1	4	13
Lower 27%: N = 8 (IQ = 92 and below)	2	3	3	0	8
Total	12	7	5	5	29

*AR = Arithmetic Reasoning
AC = Arithmetic Computation

The above data were derived from administering the Otis Quick-Scoring Mental Ability Tests (New Edition, Beta Test, Form EM) and the Stanford Achievement Test (Intermediate Arithmetic Test, Form J).

Grade Placement at time of testing = 6.7.
Median Beta-IQ of class = 101.
Median Arithmetic Reasoning grade-equivalent score = 6.3.
Median Arithmetic Computation grade-equivalent score = 6.4.
(Total group norms)

Consider the eight children in this sixth-grade class who represented the upper 27% of the class in general intelligence, with IQ's of 113 and above. Of these eight children, we note that:

a. four were in the upper 27% of the class in achievement in both arithmetic reasoning and arithmetic computation.

b. two were in the upper 27% of the class in achievement in arithmetic reasoning but not in achievement in arithmetic computation.

c. one was in the upper 27% of the class in achievement in arithmetic computation but not in achievement in arithmetic reasoning.

d. one was not in the upper 27% of the class in achievement in either arithmetic reasoning or arithmetic computation.

Are all eight of these children "mathematically superior"? Most likely, no. Are the four children in category (a) "mathematically superior"? Likely, yes; but not necessarily so. Are the children in categories (b) and (c) "mathematically superior"? The two in (b) may possibly be such; the one in (c) is not as likely to be so. Is the child in category (d) "mathematically superior"? Probably not; but we certainly can't be sure on this evidence alone.

But what about the other 21 children in the class, those not in the upper 27% in general intelligence, with IQ's below 113? May not some of these be talented mathematically? Quite possibly, yes.

Among other things it is evident from Table I that a particular child's achievement level in arithmetic reasoning may differ from his achievement level in arithmetic computation. This difference often is of definite practical significance. Furthermore, two children may have the same *average* arithmetic achievement level but differ markedly in their respective achievement levels in arithmetic reasoning and arithmetic computation. For example: Two children in the class involved in Table I each had an average arithmetic grade score of 6.8. Their respective grade scores in arithmetic reasoning (AR) and arithmetic computation (AC) are tabulated below:

	Grade Score	
	AR	*AC*
Pupil X	7.3	6.3
Pupil Y	6.4	7.2

In a similar way, two children having the same IQ often may differ significantly in the pattern of quantitative and non-quantitative components which are involved in their "general intelligence". For this reason one of two children who are above average in IQ may be "mathematically superior" or "mathematically talented" and the other not. The higher the IQ, the less likely we may expect that this would be so. The pattern of quantitative and non-quantitative components involved in a particular child's general intelligence also accounts for the fact that he may be definitely talented mathematically but not particularly high in IQ.[1]

[1]Sometimes the IQ of a mathematically talented child has been determined by an intelligence test that is almost entirely non-quantitative in its content. In these instances the IQ very well may be lower than might have been the case if quantitative items had been included in the test.

When we take these facts into account along with our knowledge of the correlation between achievement in arithmetic reasoning and achievement in arithmetic computation and of the correlation of each of these aspects of arithmetic achievement with general intelligence, we realize the impossibility of making a highly reliable identification of the mathematically superior or talented child on these bases alone. However, IQ and level of arithmetic achievement can serve as helpful criteria in conjunction with others to be mentioned if we keep these facts in mind:

1. Although a child with a high IQ is quite likely to be mathematically talented, a high IQ is neither a sufficient nor a necessary condition. There are children with above average IQ's who are not mathematically talented, and there are mathematically talented children who do not have particularly high IQ's.
2. If a child has a high IQ *and* a high level of arithmetic achievement, he is more likely to be talented mathematically than if he is high in only one of these two factors. In this connection, a high level of achievement in arithmetic *reasoning* is of more significance than is a high level of achievement in arithmetic computation. In any event, we should not overlook the fact that some mathematically talented children have high IQ's but are *under*-achieving because of motivational, emotional, and other non-intellective factors.
3. A few of our newer arithmetic achievement tests provide a separate measure of mathematical understanding and conceptual learning, in addition to the usual measures of arithmetic reasoning and arithmetic computation. A child with a high level of achievement in this aspect of arithmetic ability, coupled with a high IQ and a high level of achievement in arithmetic reasoning, is almost certain to be mathematically talented.

What Are Other Characteristics of Mathematically Superior Children? Mathematically talented or superior children possess and exhibit other characteristics and behaviors that help us distinguish these children from those who are less capable mathematically. Some of the more important and significant of these attributes are listed below.

1. Sensitivity to, awareness of, and curiosity regarding quantity and the quantitative aspects of things within the environment.[2]
2. Quickness in perceiving, comprehending, understanding, and dealing effectively with quantity and the quantitative aspects of things within the environment.
3. Ability to think and work abstractly and symbolically when dealing with quantity and quantitative ideas.
4. Ability to communicate quantitative ideas effectively to others, both orally and in writing, and readily to receive and assimilate quantitative ideas in the same ways.
5. Ability to perceive mathematical patterns, structures, relationships, and inter-relationships.

[2]For our purposes here, expressions such as "quantity" and "quantitative" should be construed to embrace concepts of size, form, shape, position, and the like whenever applicable.

6. Ability to think and perform in quantitative situations in a flexible rather than a stereotyped manner; with insight, imagination, creativity, originality, self-direction, independence, eagerness, concentration, and persistence.
7. Ability to think and reason analytically and deductively; ability to think and reason inductively and to generalize.
8. Ability to transfer learning to new or novel "untaught" quantitative situations.
9. Ability to apply mathematical learning to social situations, to other curricular areas, and the like.
10. Ability to remember and retain that which has been learned.

It is evident immediately that the above characteristics are things that cannot be measured as easily or objectively as we measure general intelligence or arithmetic achievement. We have no standardized tests for measuring each of these characteristics as such. We have no way of expressing a child's ability in each attribute as a grade score or an age score or some index or ratio such as IQ.

Rather, these characteristics are things the teacher must seek to find evidence of in a much more informal and sometimes subjective way as she observes a child's day-to-day behavior. This evidence is seen in a wide variety of ways and forms and situations. At times it is quite obvious that a child possesses one or another of the characteristics listed. Often, however, the teacher must look and probe beneath the surface performance of a child to catch glimpses of evidence of some of these attributes.

It is important for us to recognize that the characteristics we have listed are not all-or-none attributes. This statement can and should convey at least two interpretations or implications. On the one hand, we should understand that each of the listed characteristics is not something which a child either has or does not have. Rather, each characteristic is something that will differ in degree among children of varying mathematical abilities. The mathematically talented child generally will possess a particular characteristic to a rather high degree; the less capable child very well may exhibit some evidence of that characteristic, but to a lesser degree. On the other hand, we also should understand that it is not a matter of a child's possessing either all of the listed attributes or none of them. A child may possess some of these characteristics to a rather high degree and others to a much lower degree. The mathematically superior child generally will possess most, if not all, of the listed characteristics to a relatively high degree.

Intelligence, arithmetic achievement, and additional characteristics such as the ten listed previously, are major factors we must take into account as we attempt to identify the mathematically superior child. We obviously have no way of quantifying and weighting all of these factors and then combining them into a mathematical equation that will tell us whether or not a particular child is mathematically talented. Rather, we must take into account as best we can the pattern of evidence we get in one way or another regarding a child's possession of these various characteristics. As a child displays more and more of these attributes to a

relatively high degree, we can become increasingly confident that he is mathematically talented. If perchance we err in our judgment of any child, it would be much less damaging to him if we had judged him to be talented when he is not than if we had not judged him to be talented when he is.

How Shall We Provide for the Mathematically Superior Child? Let us assume that certain children have been identified as mathematically talented. The question that then arises is obvious: How shall we provide for them effectively? The answer to that question is far from obvious, however!

For one thing, the problem of providing effectively for the mathematically superior elementary school child is in part an *administrative* problem. Shall we provide for the mathematically talented child within the framework of the regular heterogeneous classroom, or shall we adopt a plan of homogeneous grouping and set up special classes for mathematically superior children?

Some schools, of course, are too small to follow a plan of homogeneous grouping by classes even if they wished to do so. In the case of schools large enough to permit homogeneous grouping by classes, we are aware of the many well-known pros and cons and the sharp differences of opinion regarding such an administrative practice.[8]

Actually, as far as the present monograph is concerned, the issue that has been raised is largely irrelevant or, if not irrelevant, it is unimportant. The materials illustrated in Part II of this monograph would be useful in either administrative setup. Although the materials were developed in relation to their use in regular heterogeneous classrooms, they could be used equally well with children who were grouped homogeneously in special classes.

Which Shall It Be, Acceleration or Enrichment? As pointed out in the previous section of this monograph, the problem of providing effectively for the mathematically superior elementary school child is in part an administrative problem. Much more importantly, however, it is a *curricular* problem. The crucial aspects of the problem are to be found in the content and methods and materials of instruction.

The question we face and must answer may be stated in this way: Can we provide most effectively for the mathematically talented child in the elementary school by an *accelerated* instructional program or by an *enriched* instructional program? (Or possibly by an instructional program that is a combination of both acceleration and enrichment?)

Discussions of this question frequently have resulted in confusion and misunderstanding for semantic reasons, stemming from differing connotations of the terms "acceleration" and "enrichment." This semantic difficulty will be evident in portions of the material which follows. It is hoped, however, that the intended interpretation will be seen clearly.

Glennon has pointed out that ". . . there are two teaching practices for the organization and presentation of arithmetic material. The first I

[8]If we were to adopt a plan of homogeneous grouping by classes, we must recognize that some children who are mathematically talented are not talented in other curricular areas.

will call *vertical* enrichment (without grade-skipping) and the second *horizontal* enrichment. [Many persons would elect to use the terms *acceleration* and *enrichment*, respectively, to identify these same two teaching practices.[4]] Vertical enrichment [acceleration] would consist of helping the child proceed through the arithmetic program of the particular grade finishing perhaps in a fraction of the time required by the normal child for whom the book was written. Having completed this program, the teacher then would provide him with the arithmetic program of the next grade — with the teaching, whenever necessary, consisting of individualized instruction. The danger of vertical enrichment [acceleration] is that the child will proceed, at his usual rapid pace, through a very *narrow curriculum* making very few of the interesting side trips that are available. He could be compared to the driver of a racing car — making great speed but seeing little of what is around him." (B-13: 41)†

It is important to note in this connection that acceleration often becomes a *one-dimensional* form of provision for individual differences; i.e., differentiation solely in terms of rate of learning or rate of progress. In another source one of the authors of the present monograph has pointed to the fact that ". . . adjustment of rate of progress *alone* is not an adequate provision for individual differences in arithmetic. It is of little value to permit children to progress at their own rates through uniform experiences and assignments in a uniform way. Various other adjustments are equally necessary." (B-46: 301)

Glennon then goes on to say: "More appropriate for this gifted child in the elementary school would be a program of horizontal enrichment. This would provide the learner with the opportunity of making many side trips into new and challenging territory. The side trips would give him insight into the relationships that exist between and among the less frequently used aspects of this body of knowledge called arithmetic. By so doing he would greatly increase his perception of arithmetic as a system of related ideas — as an area the topology of which the average child rarely sees." (B-13: 41)

Morton advocates a similar approach to the problem. He says: "Rather than provide for the acceleration of these [more capable] pupils and their more rapid progress through the regular work of the school years, we suggest that they be given, at each grade level in the elementary school, the junior high school, and the senior high school, exercises similar to those which other pupils have but more challenging. These pupils should rise to higher levels of performance than can be expected of other pupils.

"Special exercises for pupils who excel should be limited largely to those which develop greater degrees of mathematical insight and understanding and which lead to worthwhile skills.

"The 'grouping-in-depth' plan has been followed successfully at elementary school and junior high school levels. This plan does not provide for acceleration, that is, for studying at any grade level the topics which are normally studied at a higher grade level. The 'grouping-in-depth' plan may

[4]Glennon uses the term *acceleration* to refer to the administrative practice of "grade-skipping."

†This designation means page 41 of reference B-13 in the bibliography.

well be supplemented and followed by an acceleration plan, however. In the senior high school, it may be desirable that those who excel embark on an accelerated program." (B-33: 121, 122)

Junge has emphasized that this approach to provision for individual differences in arithmetic differs from the approach usually taken in an area such as reading. She says: "In general, it is agreed that there will be little, if *any differentiation of topics in arithmetic for children of different abilities.* There is a *core of understanding* and *skills* needed by all children. *Differentiation and enrichment* will therefore be *not in topics*, but in *levels of learning, in depth, and in scope.* The amount of concrete background in any topic can be varied, rates of learning can be varied, and extent of a topic can be varied. Teaching so as to make variations in depth and scope is made easier by grouping. However, *grouping* in arithmetic is carried on in a fashion different from that in reading. In reading, learning is carried on in a 'ragged front.' In a fourth grade, for example, one group of children will be reading from fourth grade materials, another may be reading third grade materials and still another group may be reading from source materials rated fifth or sixth grade in reading level. However in arithmetic, the learning is carried on in an 'even front.' All pupils in class work on the same area or concept but distinct provision is made for different levels of work in the area. . . . All children are working on developing an understanding of the same concept [for example], but the able child is working at a level which challenges his thinking." (B-27: 144, 145)

Moser has expressed the belief that: "Without question, the development of the concept of teaching by levels of learning has become the most important development in arithmetic instruction in the past decade. Indeed its full implications in terms of curriculum development are yet to be explored." (B-34: 225)

[Interested readers will find specific illustrations of this levels-of-learning idea in operation in discussions by Clark (B-7), Junge (B-27), Morton (B-33), Moser (B-34), Smith (B-41), and Weaver (B-45 and B-46).]

It is obvious that the authors of this present monograph believe that the key to effective provision for the more capable child mathematically is to be found in a program of *enrichment* rather than in a program of acceleration, as these terms have been used in the foregoing discussion. If the authors had favored the idea of acceleration (or vertical enrichment as Glennon describes it), Part II of the monograph never would have been developed.

How Shall We Enrich the Mathematics Program for More Capable Children? If we accept the premise that effective provision for the mathematically talented elementary school child should be based on enrichment rather than on acceleration, we then face an obvious question: What kind or form of enrichment shall be used?

An attempt to answer this question has both negative and positive aspects. First, from a somewhat negative standpoint, it is important for us to recognize these things clearly:

1. There is no *single* method of enrichment or type of enrichment that is all-sufficient. Numerous forms of activity and experience must be used.

2. Doing more of the same thing that already can be done well does not constitute effective enrichment. Some element or degree of newness or "differentness" must be involved.
3. Routine forms of activity that can be done mechanically or unthinkingly do not constitute effective enrichment. The superior mathematical ability of the child should be operative in some way.
4. An effective enrichment program is not exclusively self-directed and self-administering on the part of the pupil. It must involve careful planning, specific preparation, and systematic instruction on the part of the teacher.

Now, from a more positive standpoint, we may ask the following questions about any contemplated enrichment activity or experience in relation to the mathematically talented children with whom it is to be used:

1. *Is the activity or experience correlated with or related to the on-going arithmetic program, being an outgrowth thereof and/or contributing back thereto?*
2. *Will the activity or experience motivate the children positively and arouse or stimulate their interest, contributing toward the development of a favorable attitude toward mathematics and a desire for further mathematical activities and experiences?*
3. *Will the activity or experience represent an intellectual challenge to the children?*
4. *Will the activity or experience extend and/or deepen the children's mathematical knowledge, ability, and understanding?*
5. *Will the activity or experience increase the children's appreciation of mathematics?*

In regard to a particular enrichment activity or experience we recognize that a "yes" or "no" answer to each of the above questions often is a relative rather than an absolute matter. We also recognize that a very worthwhile enrichment activity or experience would not always satisfy each of these five criteria equally well. Generally speaking, however: the more of the preceding questions that can be answered clearly in the affirmative in relation to a particular enrichment activity or experience, the more effective it likely will be.

Our professional literature includes quite a few suggestions for enrichment activities and experiences, ranging from rather brief and general listings to much more extensive and detailed illustrations and compilations. Most of these are in the form of ideas that the teacher can translate into appropriate kinds of learning activities and experiences; a few, however, are in the form of materials that can be placed in the hands of pupils for their own more or less independent use. These various suggestions and materials differ in the extent to which they reflect the five previously mentioned criteria for effective enrichment activities and experiences. Be that as it may, the reader certainly will want to consider carefully and thoughtfully suggestions and ideas presented in one form or another by Brueckner and Grossnickle (B-3), by Brueckner, Grossnickle, and Reckzeh (B-4), by Brydegaard (B-5), by Clark and Eads (B-8), by Denny (B-10),

by Glennon (B-13), by Grossnickle (B-15), by Grossnickle and Brueckner (B-17), by Junge (B-27), by Larsen (B-28), by Marks, Purdy, and Kinney (B-29), by McSwain and Cooke (B-31), and by Spitzer (B-42 and B-43).

Many of the suggestions given in these references are quite familiar or obvious, such as having the mathematically superior child work with "more difficult and challenging examples, problems, and applications." As might be expected also, the suggestions include various types of units, projects, reports, notebooks, constructions, models, exhibits, displays, field trips, and the like.

For purposes of this monograph, however, we wish to direct particular attention to the following kinds of things that frequently serve as the basis for various forms of enrichment activity and experience.

1. Historical material pertaining to
   1.1 — number ideas and systems of notation,
   1.2 — written forms of computation (algorisms),
   1.3 — measures (units of measure and measuring instruments).

2. "Recreational" mathematics, including
   2.1 — number games, tricks, puzzles, riddles, etc.,
   2.2 — number pleasantries, oddities, etc.

3. Computational short-cuts and checking procedures (both oral and written).

4. Alternative patterns of thinking, methods of procedure, etc. (in examples and problems, both oral and written).

5. "Excursions" into simpler forms of more advanced mathematical ideas, such as
   5.1 — notational systems with non-decimal bases,
   5.2 — properties of *number* systems (as distinguished from *notational* systems),
   5.3 — theory of numbers.

Although some very helpful enrichment activities and experiences can be developed from things such as these above, a word of caution is in order. The mere fact that an activity or experience has its basis in one of the above areas is no guarantee that it necessarily will be effective as enrichment for more capable children. One example should suffice by way of illustration. Suppose we wanted to use the familiar short-cut for squaring a two-place number ending in 5 as the basis for an enrichment activity. If we merely helped children learn the *how* of the short-cut, this would be of little value as enrichment for children of superior mathematical ability. Many far less capable children can easily learn simply *what* to do. If, on the other hand, we were to give attention to the *why* of the short-cut (i.e., its underlying mathematical basis) along with the *how*, then we are much more likely to have an activity or experience that will be truly enriching for more capable children. The important thing is the *nature* of the activity or experience in terms of the kind of "thinking" involved.

What Are Other Promising Sources of Enrichment Material? As we seek ideas and suggestions for effective enrichment material for mathematically superior elementary school children there are several sources in addition to those mentioned previously that should not be overlooked.

1. The better teachers' guides or manuals accompanying children's arithmetic textbook series offer a wide variety of good enrichment activities and experiences.[5] Many of these are correlated closely with the work of the on-going arithmetic program.
2. Some of the instructional materials and units prepared in connection with current experimental mathematics programs at or above the elementary school level can be used, generally with some modification or adaptation, as the basis for effective enrichment activities and experiences for more capable elementary school children. See the following references in the bibliography at the end of this monograph: C-5, C-6, C-17, C-19, C-21, C-22, C-23.
3. There are some, *but not many*, children's books that are quantitative in one degree or another and have a special appeal to more capable children. These are included among the following references in the monograph bibliography: B-18, B-19, B-20, B-24. A single book that falls in this category but is not listed among these four references is the following: B-1.
4. The teacher with a strong background in appropriate content mathematics will be sensitive to many mathematical ideas that can be used advantageously as the basis for effective enrichment activities and experiences. Most of the references in Section C of the monograph bibliography that were not cited in 2 above are helpful in pointing up the kind of background in content mathematics that is of value in this connection.

This leads to what the authors consider to be a fitting conclusion to Part I of this monograph. It is their firm conviction that an elementary school teacher's ability to provide effectively for the mathematically superior children in her classroom is directly proportional to the strength of her own background in appropriate content mathematics. As this background is strengthened, her provisions for mathematically talented children in her classroom will be made more easily and more effectively. Try it and see!

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[5] A recent article in one of our professional periodicals attempted to illustrate the value and helpfulness of these sources. See the following: Foote, Evelyn W. "Using Teachers' Manuals for Deeper Learning." (*The Arithmetic Teacher*, 6: 17–22, February 1959.)

PART II

SELECTED SELF-ADMINISTERING PENCIL-AND-PAPER ENRICHMENT ACTIVITIES

Part I of this monograph emphasized, among other things, the fact that any effective enrichment program for mathematically superior elementary school children necessarily must involve a variety of activities and experiences. Part II of the monograph is devoted to specific illustrations of one particular kind of instructional experience.

In the over-all enrichment program there is a definite place for written (i.e., pencil-and-paper) activities that are self-administering as far as the child is concerned. Actually, however, relatively few of these are available at the present time, even as models. Although we have good reason to believe that many more of these will appear in published form in the not-too-distant future, a teacher today has little to guide her in preparing this form of enrichment material for use in her own classroom. It is for this reason that Part II of the monograph was compiled.

Seventeen illustrative self-administering pencil-and-paper enrichment activities, based on various aspects of mathematical content, have been included here in Part II. Insofar as possible, these were constructed with the criteria mentioned on page 11 of the monograph in mind. Although the activities certainly are not perfect ones, it is hoped that they are sufficiently well prepared to illustrate clearly the potential in this form of enrichment experience.

Something should be said here regarding the format in which the activities appear. Printing limitations and restrictions made it impossible for the material to be spaced, etc. in precisely the way it should be for actual use by elementary school children. Each activity had to be adjusted to one or two full pages, with no "part pages" allowed. This not only affected aspects of format such as spacing, it also meant that at times certain parts of the original activity had to be deleted or condensed in some way. Facts such as these must be kept clearly in mind as persons follow any or all of these activities as models or guides in preparing similar materials for use with children in an actual classroom situation.

Each activity can be related directly to one phase or another of the typical on-going elementary school arithmetic program. No attempt has been made in the monograph, however, to associate a particular activity with a particular grade level. In some instances this association is clearly obvious and would not need to be specified. In other instances a particular activity might be used at one of several grade levels by modifying the sizes or kinds of numbers involved, the vocabulary used, etc. The authors feel that their purpose will be accomplished better if readers do not try to associate a particular activity with a particular grade level, but instead try to see the way in which an activity can be modified or adapted for use at different grade levels.

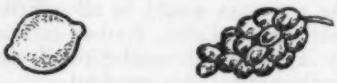
Finally, the authors want to be sure that no one will misinterpret the use of these activities in the following important respect. Although each activity is purportedly self-administering, it is not intended that any of the activities would be all-sufficient for the development of a particular mathematical idea. Rather, each activity would be preceded by or followed by appropriate teacher-pupil discussion or the like relative to the mathematical idea involved.

List of Enrichment Activities

No.	*Title of Activity*	*Page*
1.	How Many Together?	16
2.	Sets of Things	17
3.	What Is the Next Number?	18–19
4.	Triangular Numbers and Square Numbers	20
5.	Cents, Nickels, and Quarters	21
6.	How Many Squares Do You See?	22
7.	Adding Columns from Left-to-Right	23
8.	Doing Column Addition in a Different Way	24
9.	Working with Magic Squares	25
10.	Going Around in a Circle	26–27
11.	Left-to-Right Subtraction	28
12.	Subtraction with Equal Additions	29
13.	Multiplying by 9, by 99, and by 999	30
14.	Multiplying by Using Factors of the Multiplier	31
15.	Dividing by 25	32
16.	Dividing in a Different Way	33
17.	Writing Fractions As Sums of Unit Fractions	34

1. How Many Together?

This is Tom's fruit. He has 1 apple and 1 pear. Tom has ..... different kinds of fruit.



This is Bob's fruit. He has 1 lemon and 1 bunch of grapes. Bob has ..... different kinds of fruit.

This is Betty's fruit. She has 1 pear and 1 bunch of cherries. Betty has ..... different kinds of fruit.

This is Sally's fruit. She has 1 pear and 1 apple. Sally has ..... different kinds of fruit.

Look at Tom's fruit and at Bob's fruit. Tom has ..... kinds of fruit. Bob has ..... kinds of fruit. Tom and Bob together have ..... different kinds of fruit. Using numbers, we can write: 2 and 2 are ..... .

Now look at Betty's fruit and at Sally's fruit. Betty has ..... kinds of fruit. Sally has ..... kinds of fruit. But Betty and Sally together do not have 4 different kinds of fruit. They have only ..... different kinds of fruit. So, using numbers, this time we can write: 2 and 2 are ..... .

Now look at Tom's fruit and at Sally's fruit. Tom has ..... kinds of fruit and Sally has ..... kinds of fruit. Tom and Sally together have just ..... different kinds of fruit. Using numbers, we can write: 2 and 2 are ..... .

Using numbers, write the stories that tell how many different kinds of fruit these children will have:

Betty and Bob together: .....
Tom and Betty and Sally together: .....
Tom and Bob and Betty together: .....

2. Sets of Things

A group or collection of things that are alike in some way is sometimes called a *set* of things. Here are some sets of names of children in a classroom.

Set A	*Set B*	*Set C*	*Set D*
Alan Pete	Alan	Alan Edna	Edna
Jean Sara	Edna	Bret Jean	Nick
	Tony	Tony	

Set A is the group of children who have finished their reading work.
Set B is the group of children who have finished their spelling work.
Set C is the group of children who have finished their arithmetic work.
Set D is the group of children who have finished their science work.

No two children in any of these four sets have the same name. This means that Alan in Set A, for example, is the same child as Alan in Set B or Alan in Set C. This is also true for Edna, Jean, and Tony.

Which children have finished *either* their reading work (Set A) *or* their science work (Set D)? Write their names on the line below:

.....

You should have written six different names. Did you?

Which children have finished *either* their reading work (A) *or* their spelling work (B)? Write their names on the line below:

.....

You should have written just *six* different names. Which name did you have to be careful not to write twice?..... .

On the line below write the names of the children who have finished *either* their spelling work (B) *or* their arithmetic work (C):

.....

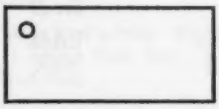
Be careful this time! On the line below write the names of the children who have finished *both* their spelling work (B) *and* their science work (D):

.....

Write the names of the children who have finished *both* their reading work (A) *and* their science work (D): .....

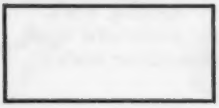
3. What Is the Next Number?

Mike learned an interesting thing about certain numbers when he put marbles in boxes in a special way.

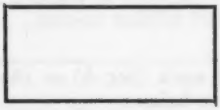
A 

1 .....

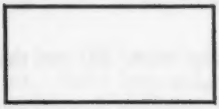
Mike found 1 marble in box A. He then put just as many marbles in *each* of the next two boxes. Draw 1 marble in box B and 1 marble in box C.

B 

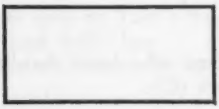
..... Write the numeral that tells how many marbles Mike now has in box B. He put just as many more marbles in *each* of the next two boxes. Draw these marbles in box C and in box D.

C 

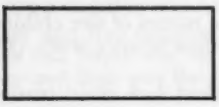
..... Write the numeral that tells how many marbles Mike now has in box C. He put just as many more marbles in *each* of the next two boxes. Draw these marbles in box D and in box E.

D 

..... Write the numeral that tells how many marbles Mike now has in box D. He put just as many more marbles in *each* of the next two boxes. Draw these marbles in box E and in box F.

E 

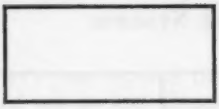
..... Write the numeral that tells how many marbles Mike now has in box E. He put just as many more marbles in *each* of the next two boxes. Draw these marbles in box F and in box G.

F 

..... Write the numeral that tells how many marbles Mike now has in box F. He put just as many more marbles in *each* of the next two boxes. Draw these marbles in box G and in box H.

G

..... Write the numeral that tells how many marbles Mike now has in box G. He put just as many more marbles in the next box, the last one he had. Draw these marbles in box H.

H  ..... Write the numeral that tells how many marbles Mike now has in box H.

Now look at the numerals that tell how many marbles there are in each box. Are these the numerals you wrote:

1 1 2 3 5 8 13 21 ?

If these are not the numerals you wrote, check back over your work, find your mistake, and correct it so that you will have the right number of marbles in each box.

Now look carefully at the correct numerals: 1, 1, 2, 3, 5, 8, 13, 21. They stand for part of a special set of numbers that follow each other in an interesting way.

Starting at the left, do you see how we can use the first two numbers (1 and 1) to tell us what the third number (2) will be? Does $1 + 1 = 2$?

Can we use the second and third numbers (1 and 2) in this same way to tell us what the fourth number (3) will be? Does $1 + 2 = 3$?

Can we use the third and fourth numbers (2 and 3) in this same way to tell us what the fifth number (5) will be? Does $2 + 3 = 5$?

Do you think we always could find the next number in the set in this way? Check each of these and see:

The sixth number in the set is 8. Does $3 + 5 = 8$?
The seventh number in the set is 13. Does $5 + 8 = 13$?
The eighth number in the set is 21. Does $8 + 13 = 21$?

We can find more numbers that follow after each other in this same way. The next number in the set after 21 would be $13 + 21$, or..... . See if you can find the next two numbers after the one you just found.

....

Do you think you could find the *last* number in this special set?

4. Triangular Numbers and Square Numbers

1 o	1 o +2 o o	1 o 2 o o +3 o o o	1 o 2 o o 3 o o o +4 o o o o	1 o 2 o o 3 o o o 4 o o o o +5 o o o o o

The examples above, with their dot pictures, can help us to learn about some interesting numbers.

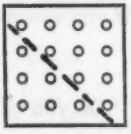
Notice that in each example except the one in the box at the left, we are to find the sum of several *consecutive* whole numbers beginning with 1. Also notice that the dot picture of each example can be drawn so that the dots form a *triangle.*

Now find the sum in each addition example at the top of the sheet. Write each sum in its usual place beneath the numbers added.

These sums, along with the number *1*, are the first five *triangular* numbers. Do you agree that 1, 3, 6, 10, and 15 are the first five triangular numbers? Could you explain why 3, 6, 10, and 15 are called triangular numbers? Can you think of why we also call 1 a triangular number?

Let's see if we can find the next triangular number after 15. We can tell what this number is by finding the sum, $1 + 2 + 3 + 4 + 5 + 6$. Find the sum. The next triangular number after 15 is ..... Draw a dot picture at the bottom of this sheet to show that this is a triangular number.

Now find the next three triangular numbers after the one you just found. Can you find a short-cut for doing this? If you can, use it. The next three triangular numbers are: ..... and ..... and .....



The dot picture at the right shows the sum of two *consecutive triangular* numbers, 6 and 10. (Notice that the dot picture for 10 has been "turned around.") The sum of 6 and 10 is 16. The dot picture for 16 is a *square*, and 16 is called a *square* number. Do you also see that 16 can be written as 4×4? We sometimes read this as "4 squared."

On the back of this sheet write four different pairs of *consecutive triangular numbers.* Find the sum of each pair. Show with a dot picture whether or not each sum is a *square* number. Whenever the sum is a square number, write it as the product of two equal factors.

What do you think are the first ten square numbers? Write them on the back of this sheet.

5. Cents, Nickels, and Quarters

Let's just suppose that we had only three different kinds of coins we could use to buy things with: cents, nickels, and quarters. Finish the statements below about these three coins:

..... cents = 1 nickel.
..... nickels = 1 quarter.

So, the nickel is worth ..... times as much as the cent, and the quarter is worth ..... times as much as the nickel.

Now let's suppose that when we buy things with these three kinds of coins, we are to pay for what we buy with *as few coins as possible.* For example: If we bought a pencil for 50¢, we should pay for it with *two quarters.* If we used any cents or nickels to pay for the pencil, we would not be using as few coins as possible.

Let's see if you could use just *three* coins to buy something that cost 31¢. Could you use 1 quarter and 1 nickel and 1 cent? ..... . With only these three kinds of coins, could you make 31¢ using fewer than three coins? ..... .

Now look at the chart below. The column at the left shows the cost of different things. In the next three columns you are to tell how many quarters, nickels, and cents you should use to pay for each thing so that you will use *as few coins as possible.* Study the examples that have been done for you, then finish the chart.

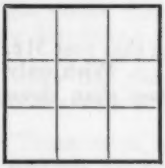
Cost	*Quarters*	*Nickels*	*Cents*
83¢	3	1	3
$1.15	4	3	0
72¢			
41¢			
$1.09			

Using costs *less than* $1.25, make up a chart like this with 10 examples for one of your friends to complete. Check his work when he is finished.

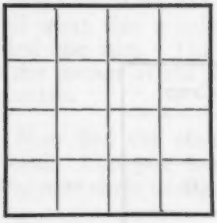
6. How Many Squares Do You See?

We call the figure at the left a ..... It has ..... equal sides and ..... equal angles. Tell your teacher or one of your friends some other things you know about this kind of figure.

There is more than one square in the drawing at the left. How many squares do you see? If you see only four squares you have not looked carefully enough. Did you forget that the large block is a square too? So, there really are ..... squares in the drawing at the left!



Look carefully at the drawing at the left. Write the number of squares you see: ..... . Did you see just 10 squares: the 9 little squares and the big block that is a square? But there are still some more squares in the drawing! There are some squares that are *two* blocks long and *two* blocks wide. If you need to, use different colors of crayon and trace each of these four-block squares. Now write the number of squares you see *in all:* ..... .



The drawing at the left is still harder. Look at it very carefully. There are some little one-block squares. There are some four-block squares. Are there any *nine*-block squares? Is there one larger square?

Write the number of squares you see *in all:* ..... .

Check your answer with your teacher. She will tell you whether or not you are right.

On the back of this sheet draw a large *triangle.* Draw several lines inside this triangle to make some smaller triangles. Then count carefully the number of triangles there are *in all.* Show this figure to one of your friends and see if he can find all the triangles you say there are.

7. Adding Columns from Left-to-Right

Did you ever think about adding a column of figures from left-to-right instead of from right-to-left? Here is one way it might be done:

```
  287
  953
+ 638
 1700   (The sum of the hundreds' column is 17 hundreds, or 1700.)
  160   (The sum of the tens' column is 16 tens, or 160.)
+  18   (The sum of the ones' column is 18 ones, or 18.)
 1878   (The sum of these partial sums is 1878.)
```

Here are some questions about this example for you to think about and be ready to answer and explain to your teacher:

1. Would it make any difference in your answer which column you added first, which column you added second, and which column you added third? Be able to "prove" your answer!

2. Would it be all right to omit the zeros in 1700 and 160 if you were sure to place just the 17 and the 16 in the right columns?

3. Could you add this left-to-right way if you had two numbers to be added instead of three? If you had more than three numbers to be added?

4. Could you add this left-to-right way if you had just 2-place numbers to be added? If you had numbers of 4 or more places?

5. When you worked the original example you added the columns from left-to-right. How did you add the partial sums: from left-to-right or from right-to-left? In the above example, would it be any easier to add the partial sums one way than the other?

6. When would you have to be very careful if you added the partial sums from left-to-right?

Now work the examples below the left-to-right way, writing each of the partial sums. (You may omit certain of the zeros if you wish.) Check each final sum by one of the checking methods you have learned.

587	269	405	3184	2957	3708
+296	841	732	+5719	7254	4993
	+635	970		+5806	7524
		+549			+6057

8. Doing Column Addition in a Different Way

$$\begin{array}{r} 25 \\ +38 \\ \hline \end{array}$$

When we add examples like those at the left, it sometimes is easier for us to think in a way that is different from our usual method.

In the example in the box we are to add 38 to 25, and we know that 38 has the parts, 30 and 8. So, we can think this way: $25 + 30 = 55$; $55 + 8 = 63$. Check this answer by working the example the usual way.

$$\begin{array}{r} 47 \\ 29 \\ +58 \\ \hline \end{array}$$

Now let's see how we can use this new way of thinking to add the example in the box at the left. Remember that 29 has the parts 20 and .....; and 58 has the parts ..... and 8. Think this way:

$47 + 20 = 67,$
$67 + 9 = 76,$
$76 + 50 = 126,$
$126 + 8 = 134.$

Now add the example in the usual way. Is 134 the correct sum?

$$\begin{array}{r} 65 \\ 82 \\ +49 \\ \hline \end{array}$$

Write in the spaces below what you would think when working the example at the left this new way:

$65 + 80 = ...,$
$..... + ... = ...,$
$..... + 40 = ...,$
$..... + ... = ...$

Check your work by finding the sum in the usual way.

$$\begin{array}{r} 426 \\ 354 \\ +120 \\ \hline \end{array}$$

Finish these steps for the example at the left:

$426 + 300 = ...,$
$..... + 50 = ...,$
$..... + 4 = ...,$
$..... + 100 = ...,$
$..... + 20 = ...$

Check your work by finding the sum in the usual way.

For each of the following examples, write out the steps in your thinking when finding the sum the new way. Check each answer.

63	37	89	541	275	562	784	298
58	96	68	235	458	821	692	525
+71	+54	+35	+472	+609	+370	+543	+769

9. Working with Magic Squares

	1	
3		
		2

You already have done some work with magic squares, so this should be easy! Use each of the numbers from 1 to 9 just once, and make a magic square in which the sum of each row, each column, and each diagonal is the same number. The square has been started for you. Finish it.

The "magic sum" is ..... .

2		
		3
	1	

Again you are to make a magic square using each of the numbers from 1 to 9 just once. Finish the work that has been started for you. This time the magic sum is ..... .

Look carefully at the two magic squares you have made. Do you see any way in which the rows and columns of the two magic squares are related to each other?

12		

Look back to the *first* magic square you made. *Add 4* to the number in each block in the first magic square and put the sum in the same block at the left.

Do you still have a magic square? ..... . If so, what is the magic sum? ..... . Can you explain how this is related to the first magic sum, and why?

24		

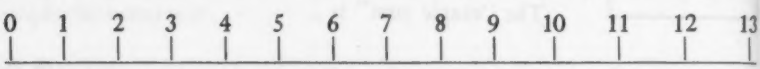
Again look back to the *first* magic square you made. This time *multiply* the number in each block of the first magic square *by 3* and put the product in the same block at the left. Do you still have a magic square? ..... . If so, what is the magic sum? ..... . Can you explain how this is related to the first magic sum, and why?

Try these same things with other magic squares. Add some other number to each part of a magic square and see if you still have a magic square. Multiply each part of a magic square by some number and see if you still have a magic square. Try to make a "rule" about changing magic squares and the change in the magic sum.

10. Going Around In A Circle

You already know a good bit about adding numbers. However, this exercise may help you to see some new ideas about addition, or it may help you to see some familiar things in new ways.

You have learned how to add numbers in different ways. One of these is by using what often is called a "number-line," as shown below.



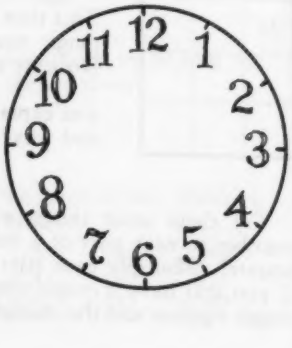
If we wanted to do an addition example such as $5 + 7$ on our number-line, we would begin at point 5 and then move 7 points *forward* — to the *right*. We then would be at point ..... . So, $5 + 7 =$ ..... .

If we wanted to do an addition example such as $9 + 8$, we would need to make our number-line go farther to the right than it now is. We can do this, of course, making it go to the right just as far as we need to. To add rather large numbers — such as $783 + 695$ — we really would have to have a *very long* number-line, wouldn't we!

Now, let's see how we can use some of these ideas about adding in a little different way. In the work you will do now, you still will add by counting *forward* in the direction of the larger numbers. However, the "number-line" you use will be a bit different from the one you used above. Before you know it, you'll really be "going around in a circle!"

Did you ever hear of "CLOCK ARITHMETIC"?

It's really nothing new. You do some kind of CLOCK ARITHMETIC almost every day.



Did you ever notice how different it is at times, though, when you add numbers on a clock? You don't always get the same answer as when you add numbers in the usual way by "regular" arithmetic.

The examples you are going to do now will help you to see how we get different-from-usual answers sometimes when we add using CLOCK ARITHMETIC.

Use the clock at the bottom of the first sheet to find the answers to these examples. Work *across* the page.

1. Pretend it is now 2 o'clock. What time will it be 5 hours *from now*? $2 + 5 =$	2. Start at 8 on the clock, then go ahead 4 spaces. At what point on the clock are you then? $8 + 4 =$	3. Find the answer to this addition example on the clock: $6 + 3 =$ .....
4. Pretend it is now 9 o'clock. What time will it be 7 hours from now? $9 + 7 =$	5. Start at 6 on the clock, then go ahead 12 spaces. At what point on the clock are you then? $6 + 12 =$	6. Find the answer to this addition example on the clock: $11 + 3 =$ .....

Now try to find the answers to these examples *without looking at the clock!* Then check your answers by "proving" them on the clock.

$7 + 4 =$	$3 + 5 =$	$9 + 8 =$	$11 + 10 =$ .....
$3 + 12 =$	$2 + 11 =$	$10 + 6 =$	$8 + 12 =$ .....
$5 + 11 =$	$12 + 10 =$	$4 + 9 =$	$2 + 8 =$ .....

How well did your answers check when you "proved" them? Draw a ring around any examples you did incorrectly the first time.

Now let's see how well you can complete the following statements to show some of the things you may have discovered about addition using CLOCK ARITHMETIC as compared with our "regular" arithmetic.

1. The answer to an addition example will be the same for both CLOCK ARITHMETIC and "regular" arithmetic when the sum of the numbers is ..... or less. The answer to an addition example will be different for CLOCK ARITHMETIC and "regular" arithmetic when the sum of the numbers is more than ..... .

2. When the same addition example has different answers for CLOCK ARITHMETIC and "regular" arithmetic, the answer for "regular" arithmetic will be ..... more than the answer for CLOCK ARITHMETIC.

3. If we add ..... to any number in CLOCK ARITHMETIC, the answer will be the same as the number we started with.

4. In CLOCK ARITHMETIC, adding 12 to a number has the same effect as adding ..... to a number in "regular" arithmetic.

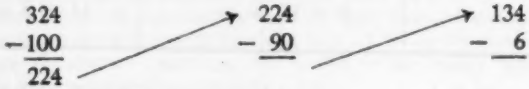
11. Left-to-Right Subtraction

You already know how to subtract from right-to-left, and understand why it seems best to work subtraction examples that way. But now let's see if we *could* subtract by starting at the left and working toward the right instead.

324 −196

Look at the example in the box at the left. It will help you to try to subtract from left-to-right if you think of 196 as made up of these parts: 100, and 90, and 6.

Do you think that 324 − 196 would be as much as 324 − 100 − 90 − 6? Make each of these three subtractions in turn and see:



Now work the original example in the box in your usual way. Is that answer the same as the answer you got by subtracting each of the parts of 196 in turn? It should be! Can you explain why?

835 −479

Here is another example for you to try the left-to-right way. First, think of 479 as made up of these parts: ....., and ....., and ..... Next, make the necessary subtractions below. Then check your answer by working the original example in the usual way.

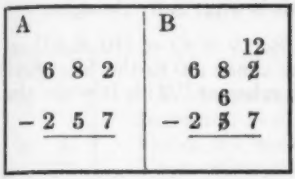
835
−400 − 70 − 9

Now you are on your own! Several more examples are given below. Do these things for each example: Copy it on the back of this sheet. Work it the new left-to-right way. Check your answer by working the original example in the usual way.

753	506	6243	8121	6035	9302
−386	−268	−2785	−3254	−4179	−5065

12. Subtraction with Equal Additions

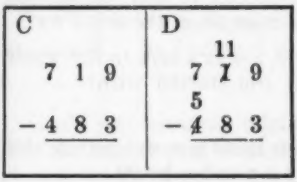
We know that in a subtraction example if the same number is added to both the minuend and the subtrahend, the difference ..... . We can use this fact to help us learn to subtract in a new way.



Look at the example in box A at the left. We cannot subtract the ones (2 − 7) unless we change the example in some way.

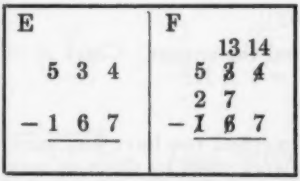
Look at how this example has been changed in box B. How many ones were added to the 2 ones in the minuend? ..... . How many tens were added to the 5 tens in the subtrahend? ..... . Does 10 ones = 1 ten? ..... . Was the same amount added to both subtrahend and minuend? ..... . How much? ..... .

Now subtract the changed example in box B. Check your answer in any way you wish. Is this true: to subtract 257 from 682, you really subtracted 267 from 692? ..... .



Now look at the example in box C at the left, and the way it has been changed in box D. This time we could subtract the ones (9 − 3) but not the tens (1 − 8).

We added ..... tens to the 1 ten and we added ..... hundred to the 4 hundreds. Does 10 tens = 1 hundred? ..... . The same amount, ... , was added to both subtrahend and minuend. To subtract 483 from 719 we really subtract ..... from ..... . Now find the answer. Check it in any way you wish.

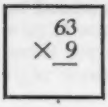


Look at the example in box E at the left, and study the way it has been changed in box F. In this example we first added ..... to both munuend and subtrahend; then we added ..... to both. Subtract the changed example. Check your answer.

Now work these examples using this new "equal additions" method.

493	817	642	790	905	500	Be sure to check
−176	−645	−387	−263	−429	−391	each answer!

13. Multiplying by 9, by 99, and by 999



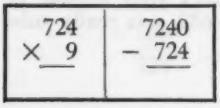
When Tom had to find the answer to the example in the box at the left, he multiplied as you usually do. Find the answer in your usual way.

Sally showed Tom another way to find the answer. She said that her way was easier. This is what Sally thought:

"I know that $9 = 10 - 1$. So, $9 \times 63 = (10 \times 63) - (1 \times 63)$. It's easy to multiply 63 by 10: just annex a 0 to the 63. And it's easy to multiply 63 by 1: it's 63. Then I subtract." This is what she wrote:

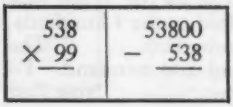
$$\begin{array}{r} 10 \times 63 = 630 \\ 1 \times 63 = \ 63 \\ \hline 9 \times 63 = 567. \end{array}$$

Does this check with your answer in the box?



Tom said he understood how Sally worked the example. He showed how to work the example in the first box at the left by writing the figures in the second box. The 7240 = ... × 724. The 724 = ... × 724. Subtract to find the answer. Check by working the example in the usual way.

Is this true: to multiply a whole number by 9, annex a zero to the whole number and then subtract the whole number you started with?..... .



The boxes at the left show how we can use this same idea to multiply a number by 99.

99 = ... − 1. So, 99 × 538 = ... × 538 − ..... × 538.

100 × 538 = ... (Did you just annex 2 zeros to the 538? .....)

1 × 538 = ... (That was easy to multiply!)

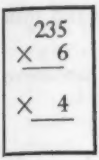
Now subtract in the second box above to find the answer. Check it in any way you wish. Does $99 \times N = 100 \times N - 1 \times N$? ..... .

Work each of these examples using the new method you have just seen. Watch out for the last two examples! You can work them by this new way if you really understand how it is used.

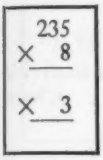
78	46	815	692	784	267	2153	4387	5216
× 9	× 9	× 9	× 9	× 99	× 99	× 99	× 999	× 999

14. Multiplying by Using Factors of the Multiplier

Often when we have to multiply by a 2-place multiplier, as in the example at the left, we can change the example so that we need to multiply only by 1-place multipliers. Let's see how we might do this.



We know that 6 and 4 are factors of 24, since $6 \times 4 = 24$. So, to find 24×235 we could find $6 \times 4 \times 235$ (or $4 \times 6 \times 235$). Finish the work in the box at the left. First find 6×235. Then multiply that product by 4.



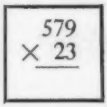
We know, too, that 8 and 3 are factors of 24: $8 \times 3 = 24$. We could check the answer we got above by first finding 8×235, and then multiplying that product by 3. Finish the work in the box at the left. Does the answer check?

Could you work the original example, or check your answer, by finding 4×235 and then multiplying that product by 6? ..... .

Could you work the original example, or check your answer, by finding 3×235 and then multiplying that product by 8? ..... .

Work each of the examples below by using factors of the multiplier. Be sure to use just 1-place numbers as factors. Check your work by using another set of factors of the multiplier, or by changing the order in which you multiply by the factors of the multiplier.

487	712	695	1253	6481	9304	7056
× 36	× 48	× 27	× 15	× 12	× 72	× 35



Look carefully at the example at the left. Be ready to explain why you can't use the method you've been working with to find the answer to this example.

15. Dividing by 25

$$475 \div \frac{100}{4} = 475 \times \frac{4}{100} = \frac{475 \times 4}{100}$$

One day, as part of an arithmetic example Sue was working, she wrote what you see in the box at the left. This suddenly gave her a bright idea! She knew that $\frac{100}{4}$ was the same as 25 in value. So, Sue decided that she had found a new way to divide a number by 25. Here is the rule she made up and then tested to see if it was right:

"To divide a number by 25, first multiply the number by 4. Then move the decimal point two places to the left to divide by 100."

$25\overline{)475}$

Let's see if Sue's rule would work for the example at the left.

First, multiply 475 by 4: $\times \begin{array}{r} 475 \\ \underline{\quad 4} \end{array}$

Now move the decimal point two places to the left to divide that product by 100. The answer is ..... . Check this answer by dividing the example in the box in the usual way.

Let's try another example, $25\overline{)791}$, using Sue's method.

First, multiply 791 by .....: $\times \begin{array}{r} 791 \\ \underline{\quad 4} \end{array}$

Next, move the decimal point in this product two places to the left to divide by 100. The answer is ..... . Check your answer by multiplying the quotient by the divisor.

Would Sue have been right if she had made up her rule this way: "To divide a number by 25, first move the decimal point in that number two places to the left; then multiply that new number by 4." ..... . Try it and see!

Work these examples using Sue's rule. Check each answer.

$25\overline{)813}$ $25\overline{)238}$ $25\overline{)947}$ $25\overline{)1216}$ $25\overline{)5604}$ $25\overline{)3967}$

16. Dividing in a Different Way

$7\overline{)460}$

Joe liked to try to discover new ways of doing things in arithmetic. One day he thought he found a new way to work the example in the box at the left. This is what Joe decided he could do:

He knew that $460 = 400 + 60$. So, he could work the two examples shown in the box at the right and then add the two quotients together to find the quotient for the original example. Do the work that Joe did, writing any remainders in fraction form as part of the quotient. Check Joe's work by doing the original example in your usual way.

$7\overline{)400}$	$7\overline{)60}$	
		+ ___

$12\overline{)780}$

At the left is another example for you to try using Joe's method. Show your work in the box at the right. Check your answer in any way you wish.

$12\overline{)700}$	$12\overline{)80}$	
		+ ___

Now try each of these examples using Joe's method. Be sure to check each answer.

$8\overline{)350}$ $6\overline{)490}$ $9\overline{)780}$ $15\overline{)240}$ *Watch this one:* $14\overline{)810}$.

These examples are harder. You will need to be careful when you make the last division in each example. Here's a hint that may help you: In the first example you must think that $456 = 400 + 50 + 6$; so you will need to make *three* divisions instead of two.

$4\overline{)456}$ $7\overline{)289}$ $8\overline{)695}$ $12\overline{)853}$ $16\overline{)538}$ $21\overline{)907}$

17. Writing Fractions As Sums of Unit Fractions

Many people of long ago did not understand as much as we do about writing common fractions. Some of these people had a way of writing only *unit fractions*,— fractions having a numerator of 1. They didn't know how to write a fraction whose numerator was 2, or 3, or more.

If these people had to try to write fractions like 3/4, they would have written it as the sum of two or more unit fractions. Using our modern number symbols, they would have written 3/4 as $1/2 + 1/4$.

Let's see how we might write some of our familiar fractions as the sum of two or more unit fractions. Suppose we wanted to write 3/8 in this way. We know that 2/8 of the 3/8 could be written as the unit fraction, 1/4. Then we would have 1/8 "left over," which is another unit fraction. So, we could write 3/8 as $1/4 + 1/8$.

Here is a harder fraction to work with: 11/16. We know that 11/16 is larger than 1/2, and that $1/2 = 8/16$. So we could use 1/2 as one of the unit fractions, and there would be 3/16 "left over." But 3/16 is not a unit fraction, so we'll have to find some unit fractions whose sum is 3/16. We know that $2/16 = 1/8$, and there would be 1/16 "left over," which is a unit fraction. So we could write 11/16 as $1/2 + 1/8 + 1/16$, using just unit fractions.

Now here are some examples for you to try by yourself. Write each of the fractions below as the sum of two or more unit fractions, but try to use as *few* unit fractions as possible for each example. This may help you understand what we mean: We could write 5/6 as $1/6 + 1/6 + 1/6 + 1/6 + 1/6$. But we also could write 5/6 as $1/2 + 1/3$. We should write it with just the two unit fractions, not the five unit fractions.

5/12 7/10 7/8 8/9 5/16 8/15 3/5

Did you discover any plan that helped you figure out which unit fractions to write each time? If so, explain it to some of your friends.

Check the unit fractions you found for each example with the unit fractions some of your friends found. Did you all have the same unit fractions? If not, explain why you got different answers.

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